Brake and Dynamometer

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A **brake** is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The capacity of a brake depends upon the following factors:

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.
**CLASSIFICATION**

- **Air brakes:**
  Air brakes are used in trucks, buses, trailers, and semi-trailers. George estinghouse first developed air brakes for use in railway service. He patented a safer air brake on March 5, 1872.

- **Hydraulic brakes:**
  The hydraulic brake is an arrangement of braking mechanism which uses brake fluid, typically containing ethylene glycol, to transfer pressure from the controlling unit. In 1918 Malcolm Lougheed developed a hydraulic brake system.

- **Electrical brake:**
  Electric brakes are used in electrically driven utilities and machines in industries and mainly in electric automotives such as electric locos. This was designed as an alternative to the conventional braking system of applying friction over the wheels to slow them.

- **Electromagnetic brakes** operate electrically, but unlike eddy current brakes, transmit torque mechanically.

- **Mechanical brake:**
  In mechanical brake friction force is applied by giving pressure on the surface of drum or disk. It may further divided axial brake and radial brake as per the direction of froce acting on drum. Radial break may internal or external.
According to shape of friction element, it may further divided in to block / shoe break, band break, disk break.

**Disc brakes:**

A brake disc is usually made of cast iron, reinforced carbon-carbon or ceramic matrix composites. This is connected to the wheel and/or the axle.

To stop the wheel, friction material in the form of brake pads (mounted on a device called a brake caliper) is forced *mechanically, hydraulically, pneumatically or electromagnetically against both sides of the disc.*
Brake Material

1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have adequate mechanical strength.
6. It should not be affected by moisture and oil.
Various geometric configurations of drum brakes are illustrated above.

Drum Brakes are classified based on the shoe geometry. Shoes are classified as being either short or long. A short shoe is one whose lining dimension in the direction of motion is so small that contact pressure variation is negligible, i.e. the
Short Shoe Analysis
For a short shoe we assume that the pressure is uniformly distributed over the contact area. Consequently the equivalent normal force $R_n = p \cdot A$, where $p$ is the contact pressure and $A$ is the surface area of the shoe.

Consequently the friction force $F_f = \mu R_n$ where $\mu$ is the co-efficient of friction between the shoe lining material and the drum material.

The torque on the brake drum is then,
$T = \mu R_n \cdot r = \mu \cdot p A \cdot r$

A quasi static analysis is used to determine the other parameters of braking.
A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. The other end of the lever is pivoted on a fixed fulcrum $O$.

(a) Clockwise rotation of brake wheel

$P =$ Force applied at the end of the lever,

$R_N =$ Normal force pressing the brake block on the wheel,

$r =$ Radius of the wheel,

$2\theta =$ Angle of contact surface of the block,

$\mu =$ Coefficient of friction, and

$F_t =$ Tangential braking force or the frictional force acting at the contact.
Self-energizing
With the direction of the drum rotation (CCW), the moment of the frictional force $\mu R_n b$ adds to the moment of the actuating force, $F_a$
As a consequence, the required actuation force needed to create a known contact pressure $p$ is much smaller than that if this effect is not present. This phenomenon of frictional force aiding the brake actuation is referred to as self-energization.

Leading and trailing shoe
For a given direction of rotation the shoe in which self energization is present is known as the leading shoe

When the direction of rotation is changed, the moment of frictional force now will be opposing the actuation force and hence greater magnitude of force is needed to create the same contact pressure. The shoe on which this is prevailing is known as a trailing shoe
Self Locking
At certain critical value of $\mu.b$ the term $(a - \mu.b)$ becomes zero. i.e no actuation force need to be applied for braking. This is the condition for self-locking.

Self-locking will not occur unless it is specifically desired.

General Procedure of Analysis
• Estimate or determine the distribution of pressure on the frictional surfaces.
• Find the relation between the maximum pressure and the pressure at any point
• For the given geometry, apply the condition of static equilibrium to find the actuating force, torque and reactions on support pins etc.
Internal Expanding Brake

An internal expanding brake consists of two shoes $S_1$ and $S_2$ as shown in Fig. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal.

Each shoe is pivoted at one end about a fixed fulcrum $O_1$ and $O_2$ and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum.

The shoes are normally held in off position by a spring. The drum encloses the entire mechanism to keep out dust and moisture.

This type of brake is commonly used in motor cars and light trucks.
It may be noted that for the clockwise direction, the right hand shoe is known as *leading or primary shoe* while the left hand shoe is known as *trailing or secondary shoe*.
Let 

\[ r = \text{Internal radius of the wheel rim}, \]
\[ b = \text{Width of the brake lining}, \]
\[ p_1 = \text{Maximum intensity of normal pressure}, \]
\[ p_N = \text{Normal pressure}, \]
\[ F_1 = \text{Force exerted by the cam on the leading shoe, and} \]
\[ F_2 = \text{Force exerted by the cam on the trailing shoe.} \]

Consider a small element of the brake lining \( AC \) subtending an angle \( \delta \theta \) at the centre. Let \( OA \) makes an angle \( \theta \) with \( OO_1 \) as shown in Fig.

the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about \( O_1 \), therefore the rate of wear of the shoe lining at \( A \)
will be proportional to the radial displacement of that point. The rate of wear of the shoe directly as the perpendicular distance from \( O_1 \) to \( OA \), i.e. \( O_1 B \). From the geometry of

From the geometry of

\[ O_1 B = O_2 O_1 \sin \theta \]

and normal pressure at \( A \),

\[ p_N \propto \sin \theta \quad \text{or} \quad p_N = p_1 \sin \theta \]

\[ \therefore \]

Normal force acting on the element,

\[ \delta R_N = \text{Normal pressure} \times \text{Area of the element} \]

\[ = p_N (b.r \delta \theta) = p_1 \sin \theta (b.r \delta \theta) \]

and braking or friction force on the element,

\[ \delta F = \mu \times \delta R_N = \mu p_1 \sin \theta (b.r \delta \theta) \]

\[ \therefore \]

Braking torque due to the element about \( O \),

\[ \delta T_B = \delta F \times r = \mu p_1 \sin \theta (b.r \delta \theta) r = \mu p_1 b r^2 (\sin \theta \delta \theta) \]
total braking torque about $O$ for whole of one shoe,

$$T_B = \mu p_1 b r^2 \Theta_2 \sin \theta \, d\theta = \mu p_1 b r^2 \left[ -\cos \theta \right]_{\Theta_1}^{\Theta_2} = \mu p_1 b r^2 (\cos \Theta_1 - \cos \Theta_2)$$

Moment of normal force $\delta R_N$ of the element about the fulcrum $O_1$,

$$\delta M_N = \delta R_N \times O_1 B = \delta R_N (O_1 \sin \theta)$$

$$= p_1 \sin \theta (b_r \delta \theta) (O_1 \sin \theta) = p_1 \sin^2 \theta (b_r \delta \theta) O_1$$

\[ \therefore \] Total moment of normal forces about the fulcrum $O_1$,

$$M_N = \int_{\Theta_1}^{\Theta_2} p_1 \sin^2 \theta (b_r \delta \theta) O_1 = p_1 b_r O_1 \int_{\Theta_1}^{\Theta_2} \sin^2 \theta \, d\theta$$

$$= p_1 b_r O_1 \int_{\Theta_1}^{\Theta_2} \frac{1}{2} (1 - \cos 2\theta) \, d\theta$$

\[ \because \sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta) \]

$$= \frac{1}{2} p_1 b_r O_1 \left[ \theta - \frac{\sin 2\theta}{2} \right]_{\Theta_1}^{\Theta_2} = \frac{1}{2} p_1 b_r O_1 \left[ \frac{\theta_2}{2} - \frac{\sin 2\theta_2}{2} - \frac{\theta_1}{2} + \frac{\sin 2\theta_1}{2} \right]$$

$$= \frac{1}{2} p_1 b_r O_1 \left[ (\theta_2 - \theta_1) + \frac{1}{2} (\sin 2\theta_1 - \sin 2\theta_2) \right]$$
Moment of frictional force $\delta F$ about the fulcrum $O_1$,

$$\delta M_F = \delta F \times AB = \delta F (r - OO_1 \cos \theta) \quad \ldots \quad (\because AB = r - OO_1 \cos \theta)$$

$$= \mu p_1 \sin \theta (b_r \cdot \delta \theta) \left( r - OO_1 \cos \theta \right) = \mu p_1 b_r (r \sin \theta - OO_1 \sin \theta \cos \theta) \delta \theta$$

$$= \mu p_1 b_r \left( r \sin \theta - \frac{OO_1}{2} \sin 2 \theta \right) \delta \theta \quad \ldots \quad (\because 2 \sin \theta \cos \theta = \sin 2\theta)$$

\[ \therefore \text{ Total moment of frictional force about the fulcrum } O_1, \]

$$M_F = \mu p_1 b_r \int_{\theta_1}^{\theta_2} \left( r \sin \theta - \frac{OO_1}{2} \sin 2 \theta \right) d\theta$$

$$= \mu p_1 b_r \left[ -r \cos \theta + \frac{OO_1}{4} \cos 2 \theta \right]_{\theta_1}^{\theta_2} = \mu p_1 b_r \left[ -r \cos \theta_2 + \frac{OO_1}{4} \cos 2 \theta_2 + r \cos \theta_1 - \frac{OO_1}{4} \cos 2 \theta_1 \right]$$

Now for leading shoe, taking moments about the fulcrum $O_1$,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum $O_2$,

$$F_2 \times l = M_N + M_F$$

Note: If $M_F > M_N$, then the brake becomes self locking.
Example

The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at ‘C’ is shown in Fig. The distance ‘CO’ is 75 mm, O being the centre of the drum. The internal radius of the brake drum is 100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm.

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of ‘θ’ with OC and may be taken as equal to $p_1 \sin \theta$, where $p_1$ is the maximum intensity of normal pressure.

The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force $Q$ required to operate the brake when 1. The drum rotates clockwise, and 2. The drum rotates anticlockwise.

Solution. Given: $OC = 75 \text{ mm}$; $r = 100 \text{ mm}$;

$\theta_2 = 135° = 135 \times \frac{\pi}{180} = 2.356 \text{ rad}$; $\theta_1 = 45° = 45 \times \frac{\pi}{180} = 0.786 \text{ rad}$; $l = 150 \text{ mm}$;

$\mu = 0.4$; $T_B = 21 \text{ N-m} = 21 \times 10^3 \text{ N-mm}$
1. Force \( Q \) required to operate the brake when drum rotates clockwise

We know that total braking torque due to shoe (\( T_B \)),

\[
21 \times 10^3 = \mu \cdot p_1 \cdot b \cdot r^2 \left( \cos \theta_1 - \cos \theta_2 \right)
\]

\[
= 0.4 \times p_1 \times b \times (100)^2 \left( \cos 45^\circ - \cos 135^\circ \right) = 5656 \quad p_1 \cdot b
\]

\[
\therefore \quad p_1 \cdot b = 21 \times 10^3 / 5656 = 3.7
\]

Total moment of normal forces about the fulcrum \( C \),

\[
M_N = \frac{1}{2} p_1 \cdot b \cdot r \cdot OC \left[ (\theta_2 - \theta_1) + \frac{1}{2} (\sin 2\theta_1 - \sin 2\theta_2) \right]
\]

\[
= \frac{1}{2} \times 3.7 \times 100 \times 75 \left[ (2.356 - 0.786) + \frac{1}{2} (\sin 90^\circ - \sin 270^\circ) \right]
\]

\[
= 13875 \times (1.57 + 1) = 35660 \text{ N-mm}
\]

and total moment of friction force about the fulcrum \( C \),

\[
M_F = \mu \cdot p_1 \cdot b \cdot r \left[ r (\cos \theta_1 - \cos \theta_2) + \frac{OC}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]
\]

\[
= 0.4 \times 3.7 \times 100 \left[ 100 (\cos 45^\circ - \cos 135^\circ) + \frac{75}{4} (\cos 270^\circ - \cos 90^\circ) \right]
\]

\[
= 148 \times 141.4 = 20930 \text{ N-mm}
\]
Taking moments about the fulcrum \( C \), we have
\[
Q \times 150 = M_N + M_F = 35660 + 20930 = 56590
\]
\[
\therefore \quad Q = \frac{56590}{150} = 377 \text{ N} \quad \text{Ans.}
\]

2. Force \( Q \) required to operate the brake when drum rotates anticlockwise

Taking moments about the fulcrum \( C \), we have
\[
Q \times 150 = M_N - M_F = 35660 - 20930 = 14730
\]
\[
\therefore \quad Q = \frac{14730}{150} = 98.2 \text{ N} \quad \text{Ans.}
\]
Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to
1. the rear wheels only,
2. the front wheels only, and
3. all the four wheels.

In all the above mentioned three types of braking, it is required to determine the retardation on of the vehicle when brakes are applied.

Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle.

The inertia force is equal and opposite to the braking force causing retardation.
Let

$\alpha = \text{Angle of inclination of the plane to the horizontal,}$

$m = \text{Mass of the vehicle in kg (such that its weight is } m.g \text{ newtons),}$

$h = \text{Height of the C.G. of the vehicle above the road surface in metres,}$

$x = \text{Perpendicular distance of C.G. from the rear axle in metres,}$

$L = \text{Distance between the centres of the rear and front wheels (also called wheel base) of the vehicle in metres,}$

$R_A = \text{Total normal reaction between the ground and the front wheels in newtons,}$

$R_B = \text{Total normal reaction between the ground and the rear wheels in newtons,}$

$\mu = \text{Coefficient of friction between the tyres and road surface, and}$

$a = \text{Retardation of the vehicle in m/s}^2$. 
1. When the brakes are applied to the rear wheels only

Motion of vehicle up the inclined plane and brakes are applied to rear wheels only.
Let \( F_B \) = Total braking force (in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value is \( \mu R_B \).

Resolving the forces parallel to the plane,
\[
F_B + m.g \cdot \sin \alpha = m.a
\]  \( \ldots (i) \)

Resolving the forces perpendicular to the plane,
\[
R_A + R_B = m.g \cos \alpha
\]  \( \ldots (ii) \)

Taking moments about \( G \), the centre of gravity of the vehicle,
\[
F_B \times h + R_B \times x = R_A \cdot (L - x)
\]  \( \ldots (iii) \)

Substituting the value of \( F_B = \mu R_B \), and \( R_A = m.g \cos \alpha - R_B \) [from equation \( (ii) \)]
\[
\mu R_B \times h + R_B \times x = (m.g \cos \alpha - R_B) \cdot (L - x)
\]
\[
R_B \cdot (L + \mu h) = m.g \cos \alpha \cdot (L - x)
\]
\[
\therefore R_B = \frac{m.g \cos \alpha (L - x)}{L + \mu h}
\]
and

\[ R_A = m \cdot g \cos \alpha - R_B = m \cdot g \cos \alpha - \frac{m \cdot g \cos \alpha (L - x)}{L + \mu \cdot h} \]

\[ = \frac{m \cdot g \cos \alpha (x + \mu \cdot h)}{L + \mu \cdot h} \]

We know from equation (i),

\[ a = \frac{F_B + m \cdot g \sin \alpha}{m} = \frac{F_B}{m} + g \sin \alpha = \frac{\mu \cdot R_B}{m} + g \sin \alpha \]

\[ - \frac{\mu \cdot g \cos \alpha (L - x)}{L + \mu \cdot h} + g \sin \alpha \]

\[ \ldots \text{(Substituting the value of } R_B) \]

Notes: 1. When the vehicle moves on a level track, then \( \alpha = 0 \).

\[ \therefore \quad R_B = \frac{m \cdot g (L - x)}{L + \mu \cdot h} \quad \text{and} \quad R_A = \frac{m \cdot g (x + \mu \cdot h)}{L + \mu \cdot h} \]

2. If the vehicle moves down the plane, then equation (i) becomes

\[ F_B - m \cdot g \sin \alpha = m \cdot a \]

\[ \therefore \quad a = \frac{F_B}{m} - g \sin \alpha = \frac{\mu \cdot R_B}{m} - g \sin \alpha = \frac{\mu \cdot g \cos \alpha (L - x)}{L + \mu \cdot h} - g \sin \alpha \]
2. When the brakes are applied to front wheels only

Motion of the vehicle up the inclined plane and brakes are applied to front wheels only.
It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.

Let $F_A = \text{Total braking force (in newtons)}$ acting at the front wheels due to the application of brakes. Its maximum value is $\mu R_A$.

Resolving the forces parallel to the plane,

$$F_A + m.g \sin \alpha = m.a \quad \ldots (i)$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m.g \cos \alpha \quad \ldots (ii)$$
Taking moments about $G$, the centre of gravity of the vehicle,

\[ F_A \times h + R_B \times x = R_A (L - x) \]

Substituting the value of $F_A = \mu R_A$ and $R_B = m g \cos \alpha - R_A$ \[ \text{[from equation (ii)]} \]

\[ \mu R_A \times h + (m g \cos \alpha - R_A) x = R_A (L - x) \]

\[ \therefore \quad R_A = \frac{m g \cos \alpha \times x}{L - \mu h} \]

and

\[ R_B = m g \cos \alpha - R_A = m g \cos \alpha - \frac{m g \cos \alpha \times x}{L - \mu h} \]

\[ = m g \cos \alpha \left( 1 - \frac{x}{L - \mu h} \right) = m g \cos \alpha \left( \frac{L - \mu h - x}{L - \mu h} \right) \]

We know from equation (i),

\[ a = \frac{F_A + m g \sin \alpha}{m} = \frac{\mu R_A + m g \sin \alpha}{m} \]
\[
\frac{\mu m g \cos \alpha \times x}{(L - \mu h)m} + \frac{m g \sin \alpha}{m} \ldots \text{(Substituting the value of } R_A) \\
= \frac{\mu g \cos \alpha \times x}{L - \mu h} + g \sin \alpha
\]

**Notes:**

1. When the vehicle moves on a level track, then \( \alpha = 0 \).

   \[
   R_A = \frac{m g \times x}{L - \mu h}; \quad R_B = \frac{m g (L - \mu h - x)}{L - \mu h}; \quad \text{and} \quad a = \frac{\mu g \cdot x}{L - \mu h}
   \]

2. When the vehicle moves down the plane, then equation (i) becomes

   \[
   F_A - m g \sin \alpha = m a
   \]

   \[
   a = \frac{F_A}{m} - g \sin \alpha = \frac{\mu R_A}{m} - g \sin \alpha = \frac{\mu g \cos \alpha \times x}{L - \mu h} - g \sin \alpha
   \]
3. When the brakes are applied to all the four wheels

Motion of the vehicle up the inclined plane and the brakes are applied to all the four wheels.
This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.

Let \( F_A \) = Braking force provided by the front wheels = \( \mu R_A \), and \( F_B \) = Braking force provided by the rear wheels = \( \mu R_B \).

Resolving the forces parallel to the plane,
\[ F_A + F_B + mg \sin \alpha = ma \quad \ldots (i) \]

Taking moments about \( G \), the centre of gravity of the vehicle,
\[ (F_A + F_B)h + R_B \times x = R_A (L - x) \quad \ldots (iii) \]
Taking moments about \( G \), the centre of gravity of the vehicle,
\[
(F_A + F_B)h + R_B \times x = R_A (L - x)
\] 
\[\ldots (iii)\]

Substituting the value of \( F_A = \mu R_A \), \( F_B = \mu R_B \) and \( R_B = m g \cos \alpha - R_A \)

[From equation \( (ii) \)] in the above expression,

\[
\mu (R_A + R_B)h + (m g \cos \alpha - R_A)x = R_A (L - x)
\]

\[
\mu (R_A + m g \cos \alpha - R_A)h + (m g \cos \alpha - R_A)x = R_A (L - x)
\]

\[
\mu m g \cos \alpha \times h + m g \cos \alpha \times x = R_A \times L
\]

\[
R_A = \frac{m g \cos \alpha (\mu h + x)}{L}
\]

\[
R_B = m g \cos \alpha - R_A = m g \cos \alpha - \frac{m g \cos \alpha (\mu h + x)}{L}
\]

\[
= m g \cos \alpha \left[1 - \frac{\mu h + x}{L}\right] = m g \cos \alpha \left(\frac{L - \mu h - x}{L}\right)
\]
Now from equation (i),

\[ \mu R_A + \mu R_B + mg \sin \alpha = ma \]

\[ \mu (R_A + R_B) + mg \sin \alpha = ma \]

\[ \mu mg \cos \alpha + mg \sin \alpha = ma \]  \hspace{1cm} \ldots \text{[From equation (ii)]}

\[ \therefore \quad a = g(\mu \cos \alpha + \sin \alpha) \]

Notes:

1. When the vehicle moves on a level track, then \( \alpha = 0 \).

\[ \therefore \quad R_A = \frac{mg(\mu h + x)}{L}; \quad R_B = mg\left(\frac{L - \mu h - x}{L}\right) \quad \text{and} \quad a = g \mu \]

2. If the vehicle moves down the plane, then equation (i) may be written as

\[ F_A + F_B - mg \sin \alpha = ma \]

or

\[ \mu (R_A + R_B) - mg \sin \alpha = ma \]

\[ \mu mg \cos \alpha - mg \sin \alpha = ma \]

and

\[ a = g(\mu \cos \alpha - \sin \alpha) \]
Example 19.15. A vehicle moving on a rough plane inclined at $10^\circ$ with the horizontal at a speed of $36$ km/h has a wheel base $1.8$ metres. The centre of gravity of the vehicle is $0.8$ metre from the rear wheels and $0.9$ metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when (1) The vehicle moves up the plane, and (2) The vehicle moves down the plane.

The brakes are applied to all the four wheels and the coefficient of friction is $0.5$.

Solution. Given: $\alpha = 10^\circ; \ u = 36$ km/h = $10$ m/s; $L = 1.8$ m; $x = 0.8$ m; $h = 0.9$ m; $\mu = 0.5$

Let $s =$ Distance travelled by the vehicle before coming to rest, and $t =$ Time taken by the vehicle in coming to rest.

1. When the vehicle moves up the plane and brakes are applied to all the four wheels

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$$a = g(\mu \cos \alpha + \sin \alpha)$$

$$= 9.81 \ (0.5 \cos 10^\circ + \sin 10^\circ)$$

$$= 9.81(0.5 \times 0.9884 + 0.1736) = 6.53 \text{ m/s}^2$$

for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 6.53} = 7.657 \text{ m Ans.}$$
and final velocity of the vehicle ($v$),

$$0 = u + a \cdot t = 10 - 6.53 \ t$$  
...(Minus sign due to retardation)

$$
\therefore \quad t = \frac{10}{6.53} = 1.53 \text{ s} \ \text{Ans.}
$$

2. When the vehicle moves down the plane and brakes are applied to all the four wheels

Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

$$a = g(\mu \cos \alpha - \sin \alpha)$$

$$= 9.81(0.5 \cos 10^\circ - \sin 10^\circ)$$

$$= 9.81(0.5 \times 0.9848 - 0.1736) = 3.13 \text{ m/s}^2$$

: for uniform retardation.

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 3.13} = 16 \text{ m} \ \text{Ans.}$$

and final velocity of the vehicle ($v$),

$$0 = u + a \cdot t = 10 - 3.13 \ t$$  
...(Minus sign due to retardation)

$$
\therefore \quad t = \frac{10}{3.13} = 3.2 \text{ s} \ \text{Ans.}$$
ROPE BREAK DYNAMOMETER

It is form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine.

It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine.

The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.

In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.
In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let

\[ W = \text{Dead load in newtons}, \]
\[ S = \text{Spring balance reading in newtons}, \]
\[ D = \text{Diameter of the wheel in metres}, \]
\[ d = \text{diameter of rope in metres}, \]
\[ N = \text{Speed of the engine shaft in r.p.m.} \]

Net Load on the brake = \((W - S)\) ....N

Distance move in one Revolution = \(\pi (D + d)\) ....m

Work done per revolution = \((W - S) \times \pi (D + d)\) ....Nm

Work done per minute by N revolution = \((W - S) \times \pi (D + d) \times N\) ....Nm/min

Brake Power = \(\frac{\text{Work done per minute}}{60}\) = \(\frac{(W - S) \pi (D + d) N}{60}\) ....Watt
Prony Brake Dynamometer:

A simplest form of an absorption type dynamometer is a prony brake dynamometer. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured.

The blocks are clamped by means of two bolts and nuts, as shown in Figure. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed.

The upper block has a long lever attached to it and carries a weight $W$ at its outer end.

A counter weight is placed at the other end of the lever which balances the brake when unloaded.

Two stops S-S are provided to limit the motion of the lever.
Let \( W \) = Weight at the outer end of the lever in newtons,
\( L \) = Horizontal distance of the weight \( W \)
from the centre of the pulley in metres,
\( F \) = Frictional resistance between the blocks
and the pulley in newtons,
\( R \) = Radius of the pulley in metres, and
\( N \) = Speed of the shaft in r.p.m.

the moment of the frictional resistance or torque on the shaft,
\[
T = W \cdot L = F \cdot R \text{ N-m}
\]

Work done in one revolution
\[
= \text{Torque } \times \text{ Angle turned in radians }
= T \times 2\pi \text{ N-m}
\]

\[
\therefore \text{ Work done per minute }
= T \times 2\pi \times N \text{ N-m}
\]

brake power of the engine,
\[
B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W \cdot L \times 2\pi N}{60} \text{ watts}
\]
Epicyclic-train Dynamometer

An epicyclic-train dynamometer consists of a simple epicyclic train of gears, i.e. a spur gear, an annular gear (a gear having internal teeth) and a pinion.

The spur gear is keyed to the engine shaft and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction.

The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts.

A weight \( w \) is placed at the smaller end of the lever in order to keep it in position.

If the friction of the pinion which the pinion rotates is neglected, then the tangential effort \( P \) exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.
Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2P.

This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight W at the end of the lever. The stops S, S are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum F,

\[ 2P \times a = W.L \]

or \[ P = \frac{W.L}{2a} \]

Let \( R = \) Pitch circle radius of the spur gear in metres, and \( N = \) Speed of the engine shaft in r.p.m.

\[ \therefore \text{Torque transmitted, } T = P.R \]

\[ \text{power transmitted} = \frac{T \times 2\pi N}{60} = \frac{P.R \times 2\pi N}{60} \text{ watts} \]
Belt Transmission Dynamo meter
Froude or Throneycroft Transmission Dynamometer

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt.

A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running.

A Froude or Throneycroft transmission dynamometer consists of a pulley A (driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley B (driven pulley) mounted on another shaft to which the power from pulley A is transmitted.
The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D which are mounted on a T-shaped frame.

The frame is pivoted at E and its movement is controlled by two stops S_y S.

Since the tension in the tight side of the belt \( T_1 \) is greater than the tension in the slack side of the belt \( T_2 \), therefore the total force acting on the pulley C (i.e. \( 2T_1 \)) is greater than the total force acting on the pulley D (i.e. \( 2T_2 \)).

It is thus obvious that the frame causes movement about E in the anticlockwise direction. In order to balance it, a weight W is applied at a distance L from E on the frame as shown in Fig.

Now taking moments about the pivot E, neglecting friction,

\[
2T_1 \times a = 2T_2 \times a + W \times L \quad \text{or} \quad T_1 - T_2 = \frac{W \times L}{2a}
\]

Let \( D = \) diameter of the pulley A in metres, and \( N = \) Speed of the engine shaft in r.p.m.

\[
\therefore \quad \text{Work done in one revolution} = (T_1 - T_2) \pi D N \text{ N-m}
\]

\[
\text{Work done per minute} = (T_1 - T_2) \pi DN \text{ N-m}
\]

\[
\therefore \quad \text{Brake power of the engine, B.P.} = \frac{(T_1 - T_2) \pi DN}{60} \text{ watts}
\]
**Torsion Dynamometer**

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel.

A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft ($J$), length of the shaft ($l$), diameter of the shaft ($D$) and modulus of rigidity ($C$) of the material of the shaft.

The torsion equation is,

$$\frac{T}{J} = \frac{C \theta}{l}$$

$\theta$ = Angle of twist in radians,  
$J$ = Polar moment of inertia of shaft

For a solid shaft of diameter $D$, the polar moment of inertia

$$J = \frac{\pi}{32} D^4$$

And for a hollow shaft of external diameter $D$ and internal diameter $d$,

$$J = \frac{\pi}{32} (D^4 - d^4)$$
From the above torsion equation

\[ T = \frac{CJ}{l} \times \theta = k \theta \]

where \( k = \frac{CJ}{l} \) is a constant for a particular shaft.

Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined.

The power transmitted, \( P = \frac{T \times 2\pi N}{60} \)

Since the angle of twist is measured for a small length of the shaft, therefore some magnifying device must be introduced in the dynamometer for accurate measurement.
Example  In a laboratory experiment, the following data were recorded with rope brake:
Diameter of the flywheel 1.2 m; diameter of the rope 12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

Solution. Given: \( D = 1.2 \text{ m} \); \( d = 12.5 \text{ mm} \) = 0.0125 m; \( N = 200 \text{ r.p.m.} \); \( W = 600 \text{ N} \); \( S = 150 \text{ N} \)

\[
\text{B.P.} = \frac{(W - S) \pi (D + d)N}{60} = \frac{(600 - 150) \pi (1.2 + 0.0125)200}{60} = 5715 \text{ W}
\]

\[= 5.715 \text{ kW Ans.}\]
Example  The essential features of a transmission dynamometer are shown in Fig. A is the driving pulley which runs at 600 r.p.m. B and C are jockey pulleys mounted on a horizontal beam pivoted at D, about which point the complete beam is balanced when at rest. E is the driven pulley and all portions of the belt between the pulleys are vertical. A, B and C are each 300 mm diameter and the thickness and weight of the belt are neglected. The length DF is 750 mm.

Find: 1. the value of the weight W to maintain the beam in a horizontal position when 4.5 kW is being transmitted, and 2. the value of W, when the belt just begins to slip on pulley A. The coefficient of friction being 0.2 and maximum tension in the belt 1.5 kN.

Solution. Given: \( N_A = 600 \) r.p.m.

\[ D_A = D_B = D_C = 300 \text{ mm} = 0.3 \text{ m} \]
1. Value of the weight $W$ to maintain the beam in a horizontal position

Given: Power transmitted $(P) = 4.5 \text{ kW} = 4500 \text{ W}$

Let $T_1 =$ Tension in the tight side of the belt on pulley $A$, and $T_2 =$ Tension in the slack side of the belt on pulley $A$.

$\therefore$ Force acting upwards on the pulley $C = 2T_1$ and force acting upwards on the pulley $B = 2T_2$

Now taking moments about the pivot $D,$

$W \times 750 = 2T_1 \times 300 - 2T_2 \times 300 = 600 (T_1 - T_2)$

$\therefore T_1 - T_2 = W \times 750 / 600 = 1.25 \text{ W N}$

the power transmitted $(P)$,

$4500 = \frac{(T_1 - T_2) \pi D_A N_A}{60} = \frac{1.25W \times \pi \times 0.3 \times 600}{60} = 11.78 \text{ W}$

$W = 4500 / 11.78 = 382 \text{ N Ans.}$
2. Value of \( W \), when the belt just begins to slip on \( A \)

Given: \( \mu = 0.2 ; \ T_1 = 1.5 \text{ kN} = 1500 \text{ N} \)

\[
2.3 \log \left( \frac{T_1}{T_2} \right) = \mu \theta = 0.2 \times \pi = 0.6284
\]

\[
\log \left( \frac{T_1}{T_2} \right) = \frac{0.6284}{2.3} = 0.2732 \quad \text{or} \quad \frac{T_1}{T_2} = 1.876
\]

\[
\therefore \quad T_2 = T_1 / 1.876 = 1500 / 1.876 = 800 \text{ N}
\]

Now taking moments about the pivot \( D \),

\[
W \times 750 = 2T_1 \times 300 - 2T_2 \times 300
\]

\[
= 2 \times 1500 \times 300 - 2 \times 800 \times 300
\]

\[
= 420 \times 10^3
\]

\[
\therefore \quad W = 420 \times 10^3 / 750 = 560 \text{ N} \quad \text{Ans.}
\]
Bevis-Gibson flash light torsion dynamometer

The working of this dynamometer depends upon the fact that the light travels in a straight line through air of uniform density and velocity of light is infinite.
When the shaft does not transmit any torque (i.e. at rest), a flash of light may be seen after every revolution of the shaft, as the positions of the slit do not change relative to one another as shown in Fig (b).

Now when the torque is transmitted, the shaft twists and the slot in the disc B changes its position. Due to this, the light does not reach to the eye piece as shown in Fig (c).

If the eye piece is now moved round by an amount equal to the lag of disc A, then the slot in the eye piece will be opposite to the slot in disc B as shown in Fig. (d) and hence the eye piece receives flash of light.

The eye piece is moved by operating a micrometer spindle, the angle of twist may be measured up to 1/100th of degree.

when the torque varies during each revolution as in reciprocating engines, it is necessary to measure the angle of twist at several different angular positions. For this, the discs A and B are perforated with slots arranged in the form of spiral as shown in figure.
Example

A torsion dynamometer is fitted to a propeller shaft of a marine engine. It is found that the shaft twists $2^\circ$ in a length of 20 metres at 120 r.p.m. If the shaft is hollow with 400 mm external diameter and 300 mm internal diameter, find the power of the engine. Take modulus of rigidity for the shaft material as 80 GPa.

Solution. Given: $\theta = 2^\circ = 2 \times \pi / 180 = 0.035$ rad; $l = 20$ m; $N = 120$ r.p.m.; $D = 400$ mm = 0.4 m; $d = 300$ mm = 0.3 m; $C = 80$ GPa = $80 \times 10^9$ N/m$^2$

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} (D^4 - d^4) = \frac{\pi}{32} \left[ (0.4)^4 - (0.3)^4 \right] = 0.0017 \text{ m}^4$$

and torque applied to the shaft,

$$T = \frac{C J}{l} \times \theta = \frac{80 \times 10^9 \times 0.0017}{20} \times 0.035 = 238 \times 10^3 \text{ N-m}$$

Power of the engine,

$$P = \frac{T \times 2 \pi N}{60} = \frac{238 \times 10^3 \times 2\pi \times 120}{60}$$

$$= 2990 \times 10^3 \text{ W} = 2990 \text{ kW Ans.}$$